**Chapter 5**

**Sequences and Series**

**5.6 Ratio and Root Tests**

**Section Exercises**

**Use the ratio test to determine whether  converges, where  is given in the following problems. State if the ratio test is inconclusive.**

317. 

Answer:  Converges.

319. 

Answer:  Converges.

321. 

Answer:  Converges.

323. 

Answer:  Converges.

325. 

Answer:  Ratio test is inconclusive.

327. 

Answer:  Converges.

**Use the root test to determine whether  converges, where  is as follows.**

329. 

Answer:  Diverges.

331. 

Answer:  Converges.

333. 

Answer:  Converges.

335. 

Answer:  Converges.

337. 

Answer:  by L’Hôpital’s rule. Converges.

**In the following exercises, use either the ratio test or the root test as appropriate to determine whether the series  with given terms  converges, or state if the test is inconclusive.**

339. 

Answer:  Converges by ratio test.

341. 

Answer:  Converges by root test.

343. 

Answer:  Diverges by root test.

**Use the ratio test to determine whether  converges, or state if the ratio test is inconclusive.**

345. 

Answer:  Converge.

**Use the root and limit comparison tests to determine whether  converges**.

347.  where   (*Hint:* Find limit of )

Answer: Converges by root test and limit comparison test since 

**In the following exercises, use an appropriate test to determine whether the series converges.**

349. 

Answer: Converges absolutely by limit comparison with  

351. 

Answer:  Series diverges.

353. 

Answer: Terms do not tend to zero:  since 

355.  where 

Answer:  which converges by comparison with  for 

357. 

Answer:  converges by comparison with geometric series.

359.  (*Hint:* )

Answer:  Series converges by limit comparison with 

**The following series converge by the ratio test. Use summation by parts,  to find the sum of the given series.**

361.  where  (*Hint:* Take  and )

Answer: If  and  then  and 

363. 

Answer: 

**The *k*th term of each of the following series has a factor  Find the range of for which the ratio test implies that the series converges.**

365. 

Answer: 

367. 

Answer: 

369. Let  For which real numbers  does  converge?

Answer: All real numbers  by the ratio test.

371. Suppose that  For which values of  is  guaranteed to converge?

Answer: 

373. For which values of  if any, does  converge? (*Hint:* 

Answer:  Note that the ratio and root tests are inconclusive. Using the hint, there are  terms  for  and for  each term is at least  Thus,   which converges by the ratio test for  For  the series diverges by the divergence test.

375. Let  where  is the greatest integer less than or equal to  Determine whether  converges and justify your answer.

Answer: One has    The ratio test does not apply because  if  is even. However,  so the series converges according to the previous exercise. Of course, the series is just a duplicated geometric series.

**The following *advanced* exercises use a generalized ratio test to determine convergence of some series that arise in particular applications when tests in this chapter, including the ratio and root test, are not powerful enough to determine their convergence. The test states that if  then  converges, while if  then  diverges.**

377. Let  Show that  For which  does the generalized ratio test imply convergence of  (*Hint:* Write  as a product of  factors each smaller than )

Answer:  The inverse of the  factor is  so the product is less than  Thus for   The series converges for 

**Chapter Review Exercises**

***True or False.* Justify your answer with a proof or a counterexample**.

379. If  then  converges.

Answer: false

381. If  converges, then  converges.

Answer: true

**Is the sequence bounded, monotone, and convergent or divergent? If it is convergent, find the limit.**

383. 

Answer: unbounded, not monotone, divergent

385. 

Answer: bounded, monotone, convergent, 

387. 

Answer: unbounded, not monotone, divergent

**Is the series convergent or divergent?**

389. 

Answer: diverges

391. 

Answer: converges

**Is the series convergent or divergent? If convergent, is it absolutely convergent?**

393. 

Answer: converges, but not absolutely

395. 

Answer: converges absolutely

397. 

Answer: converges absolutely

**Evaluate**

399. 

Answer:

**The following problems consider a simple population model of the housefly, which can be exhibited by the recursive formula  where  is the population of houseflies at generation  and  is the average number of offspring per housefly who survive to the next generation. Assume a starting population **

401. Find  if   and 

Answer:  

403. If  and  find  and 

Answer: 

**Student Project**

**Series Converging to  and **

1. The series



was discovered by Gregory and Leibniz in the late  This result follows from the Maclaurin series for  We will discuss this series in the next chapter.

1. Prove that this series converges.
2. Evaluate the partial sums  for 
3. Use the remainder estimate for alternating series to get a bound on the error 
4. What is the smallest value of  that guarantees  Evaluate 

Answer:

* 1. This series is an alternating series of the form  where  Since  we conclude that  Therefore,  is a decreasing sequence. Further,



Therefore, by the alternating series test, this series converges.

* 1.    
  2. 
  3. We need  Therefore, we need  which implies  .

3. The series



was discovered by Ramanujan in the early William Gosper, Jr., used this series to calculate  to an accuracy of more than  million digits in the  At the time, that was a world record. Since that time, this series and others by Ramanujan have led mathematicians to find many other series representations for  and 

1. Prove that this series converges.
2. Evaluate the first term in this series. Compare this number with the value of  from a calculating utility. To how many decimal places do these two numbers agree? What if we add the first two terms in the series?
3. Investigate the life of Srinivasa Ramanujan (1887 – 1920) and write a brief summary. Ramanujan is one of the most fascinating stories in the history of mathematics. He was basically self-taught, with no formal training in mathematics, yet he contributed in highly original ways to many advanced areas of mathematics.

Answer:

1. 

Therefore,



Therefore, by the ratio test, the series converges.

1. The first term in this series is 0.31830987844. Using this value to approximate , we estimate  by 3.14159273001. This approximation agrees with  through the first six decimal places. The sum of the first two terms in the series is 0.31830988618. Using this value to approximate , we estimate  by 3.14159265359.
2. Answers will vary.

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